Reliability Assessment of Stability of Underground Rock Caverns

Zhang Wengang

School of Civil & Environmental Engineering, Nanyang Technological University

Abstract

Conventional stability assessment of underground tunnels and caverns involves the determination of a factor of safety in which failure is assumed to occur when the load (stress) of the system exceeds the resistance. It is widely recognized that a deterministic analysis of the factor of safety gives only a partial representation of the true margin of safety, since the uncertainties in the design parameters affect the probability of failure. In this paper, a simplified procedure is proposed for evaluating the probability of stress-induced instability for deep underground rock caverns for preliminary design applications. Extensive parametric studies were carried out using the finite difference program FLAC^{3D} to determine the factor of safety for caverns of various dimensions and rock mass strength. Subsequently, a closed-formed limit state surface was determined using logarithmic regression following which Monte Carlo simulation was used to evaluate the probability of cavern failure.

The author is born in Feb. 1983, holding position of Research Associate in school of CEE of NTU.

1. Introduction

One of the major considerations in the design of an underground rock cavern is the evaluation of its stability since the excavation of the rock causes a redistribution of the stresses in the proximity of the underground opening. Various methods have been proposed to assess the cavern stability, and to assess the necessary support system to maintain the stability of the excavation. Common empirical methods include the use of rock classification systems such as the rock mass rating *RMR* [1] and *Q* methods [2]. Common numerical methods used to evaluate cavern stability can be categorized as continuum methods such as the Finite Element Method (FEM) [3] and Finite Difference Method (FDM) [4], and discontinuum methods such as the Distinct Element Method (DEM) [5] and the Discontinuous Deformation Analysis (DDA) [6]. The selection of a continuum or discontinuum approach depends on the size or scale of the discontinuities with respect to the size or scale of the problem that needs to be solved. There are no universal quantitative guidelines to determine when one method should be used instead of the other [7].

Conventional deterministic evaluation of stability of geotechnical structures and underground openings involves the use of a factor of safety *FS* which considers the relationship between the resistance *R* and the load (stress) *S*. The boundary separating the safe and failure domain is the limit state surface (boundary) defined as:

$$G(\mathbf{x}) = R - S = 0 \tag{1}$$

where $\boldsymbol{x}$ denotes the vector of the random variables. Mathematically, $R > S$ or $G(\boldsymbol{x}) > 0$ would denote a ‘safe’ domain, and $R < S$ or $G(\boldsymbol{x}) < 0$ would denote a ‘failure’ domain. For underground caverns, the limit state surface $G(\boldsymbol{x})$ is not known explicitly. Instead, it may be known only implicitly through a numerical procedure such as the finite element method. Therefore, the failure domain only can be found through repeated point-by-point numerical analyses with different input values. A closed-form limit state surface then is constructed artificially using logarithmic regression.

This paper utilizes the rock mass classification correlations and the numerical procedure known as the shear strength reduction technique to calculate the global factor of safety *FS* with regard to stress-induced instability. The logarithmic regression is used to determine an empirical equation relating *FS* to the cavern dimensions *B* and *H*, as well as the rock mass quality *Q*. Charts based on this equation are presented for preliminary design purposes. Monte Carlo simulation is implemented with the logarithmic regression model to calculate the probability of failure P_f for cavern stability.

2. Numerical model of rock cavern

The FLAC^{3D} code (Itasca) was used to carry out the stability analyses of the underground rock caverns using the shear strength reduction technique. The shear strength reduction technique is available in many commercial finite element and finite difference programs. The technique has been applied to a number of geotechnical problems including rock caverns [8, 9] and circular tunnels [10].

The procedure essentially involves repeated analyses by progressively reducing the shear strength properties until collapse occurs. For a Mohr-Coulomb material, by reducing the shear strength by a factor *F* the shear strength equation becomes:

$$\frac{\tau}{F} = \frac{c}{F} + \sigma_n \frac{\tan\phi}{F} \tag{2}$$

$$F = \frac{\tau}{c^* + \sigma_n \tan\phi^*} \tag{3}$$

where τ is the shear strength, σ_n is the normal stress, and $c^* = \frac{c'}{F}$ and $\phi^* = \arctan(\frac{\tan\phi}{F})$ are the new Mohr-Coulomb shear strength parameters. Systematic increments of *F* are performed until the finite element or finite difference model does not converge to a solution (i.e. failure occurs). The critical strength reduction value which corresponds to non-convergence is taken to be the factor of safety *FS*.

Only stress-induced failure was considered in this paper. In the FDM analyses, the three parameters that were varied were: the Tunneling Quality Index *Q* value, the cavern width *B* and cavern wall height *H*. *Q* cannot be directly used in the FLAC^{3D} calculations, though it is a commonly used quality index representing rock mass competence. In the analyses, the discontinuous nature of the rock is incorporated implicitly in the Mohr-Coulomb constitutive relationship used to represent the mass as an equivalent continuum. The rock mass properties are indirectly (through *RMR*) determined from the *Q* value by means of empirical equations as shown in Table 1. The *Q* value of each category and its corresponding Mohr-Coulomb rock properties to be used in the numerical calculation are shown in Table 2, in which the c, ϕ and E values are related to *Q* through the equations in Table 1. It should be noted that these relationships are intended to provide the initial estimates of the rock mass properties and should be used with caution in engineering design.

Table 1 Empirical equations relating *Q* with rock mass properties

Properties	**Equations**	**References**
RMR from *Q* value	$RMR = 7\ln Q + 36$	[11]
Cohesion c (MPa)	$c(MPa) = 0.005(RMR - 1)$	[1]
Friction angle ϕ (°)	$\phi = 0.5RMR + 4.5$	[1]
Deformation modulus E (GPa)	$E = E_m(GPa) = 2RMR - 100\ (RMR > 50)$ $E = E_m(GPa) = 10^{(RMR-10)/40}\ (RMR \le 50)$	[12-13]

Table 2 Rock mass properties with different *Q* values

Q	***c* (MPa)**	**ϕ (°)**	***E* (GPa)**	**Poisson's ratio**
0.1	0.07	12.23	1.37	0.35
0.4	0.14	19.29	3.09	0.35
1	0.18	22.50	4.47	0.35
4	0.22	27.35	7.81	0.20
10	0.26	30.56	11.30	0.20
40	0.30	35.41	19.75	0.16
100	0.34	38.62	28.57	0.16
400	0.38	43.47	49.95	0.16

The cross-section of the cavern and boundary conditions are shown in Fig. 1. The cavern roof arc is semi-circular and the overburden height *D* from the ground surface to the top of the side wall is 100 m. The cavern length in the longitudinal direction is assumed as 1 m to simulate plane strain conditions. Outer boundaries are located far from the cavern to minimize the boundary effects. Full-face excavation is assumed in all analyses. Table 3 lists the design parameters and the values that were considered. Input file for each FLAC^{3D} execution includes a geometry model (B and H) and a mechanical model of Q-related rock mass properties.

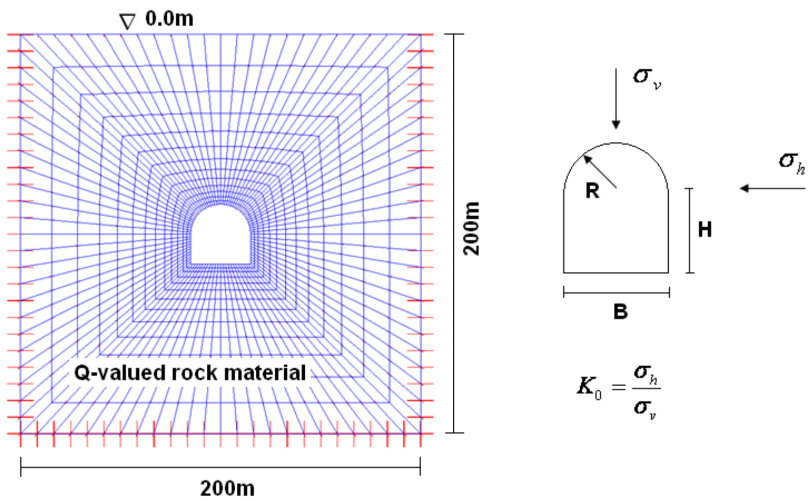


Fig. 1 Underground cavern configuration

Table 3 Input parameters for *FS* calculations

Input Parameters	Values
Tunneling quality index Q	0.1, 0.4, 1, 4, 10, 40, 100, 400
Cavern width B (m)	2, 5, 10, 15, 20, 25, 30, 40, 50
Side wall height H (m)	0.5, 1, 1.25, 2, 2.5, 3.75, 5, 6.25, 7.5, 10, 12.5, 15, 20, 25, 30, 40, 50

For each numerical analysis, the safety factor FS was determined based on the strength reduction technique. Different combinations of Q, the cavern width B and the side cavern height H were considered for three sets of ratios of B and H (B/H = 1, 2 and = 4). A total of 216 cases were considered and some results are shown in Table 4. A typical plot of Q versus FS for B = 20 m in Fig. 2 indicates the FS increases as B/H increases.

3. Determination of limit state surface using logarithmic regression

Based on the above 216 results, the logarithmic regression model was developed for predicting FS in terms of Q, B and H. The best fit equation with a coefficient of correlation R^2 = 0.96 was as follows:

$$FS_{regression} = 2.469 Q^{0.1759} B^{-0.2763} H^{-0.0973} \quad (4)$$

Table 4 Some calculations for safety factor *FS*

Q	***B* (m)**	***B/H***	***FS***	***Q***	***B* (m)**	***B/H***	***FS***
0.1	10	1	0.57	0.1	20	1	0.43
0.4	10	1	1	0.4	20	1	0.76
1	10	1	1.21	1	20	1	0.92
4	10	1	1.51	4	20	1	1.14
10	10	1	1.77	10	20	1	1.34
40	10	1	2.08	40	20	1	1.58
100	10	1	2.37	100	20	1	1.8
400	10	1	2.72	400	20	1	2.07
0.1	10	2	0.62	0.1	20	2	0.47
0.4	10	2	1.08	0.4	20	2	0.82
1	10	2	1.32	1	20	2	1.01
4	10	2	1.65	4	20	2	1.26
10	10	2	1.93	10	20	2	1.47
40	10	2	2.27	40	20	2	1.73
100	10	2	2.58	100	20	2	1.97
400	10	2	2.96	400	20	2	2.27
0.1	10	4	0.65	0.1	20	4	0.49
0.4	10	4	1.14	0.4	20	4	0.87
1	10	4	1.41	1	20	4	1.07
4	10	4	1.75	4	20	4	1.33
10	10	4	2.03	10	20	4	1.54
40	10	4	2.39	40	20	4	1.82
100	10	4	2.72	100	20	4	2.07
400	10	4	3.12	400	20	4	2.38

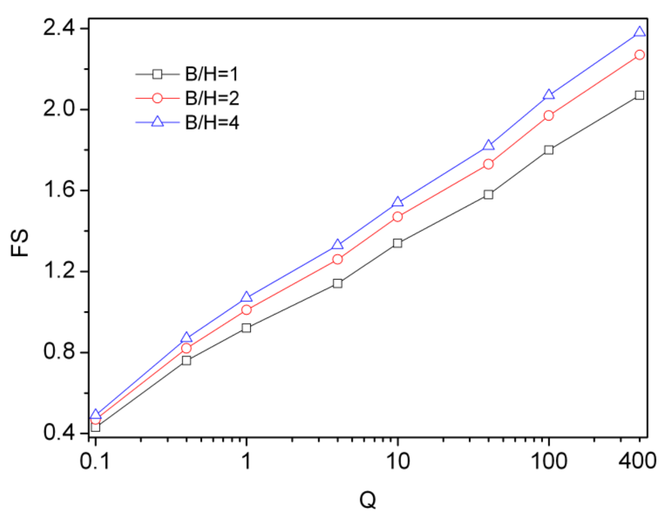


Fig. 2 Plot of *FS* vs. *Q* for *B* = 20 m

The results of the actual *FS* determined from FDM versus the predicted *FS* using Eq. (4) as shown in Fig. 3, which indicates that the predictions are in good agreement with the target *FS* values.

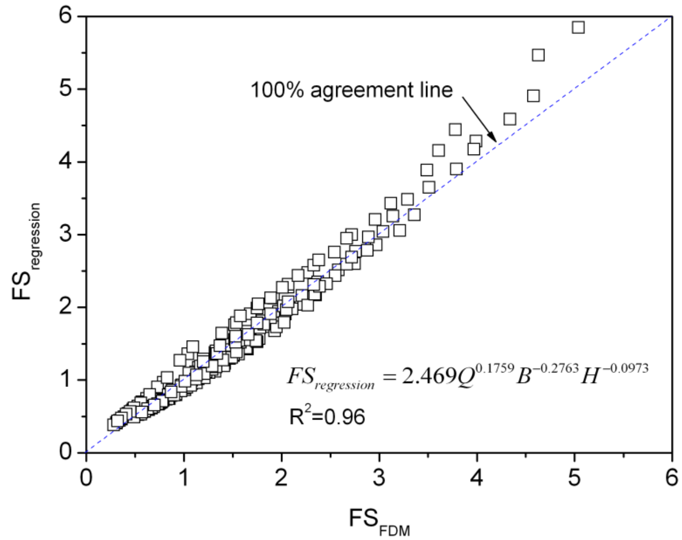
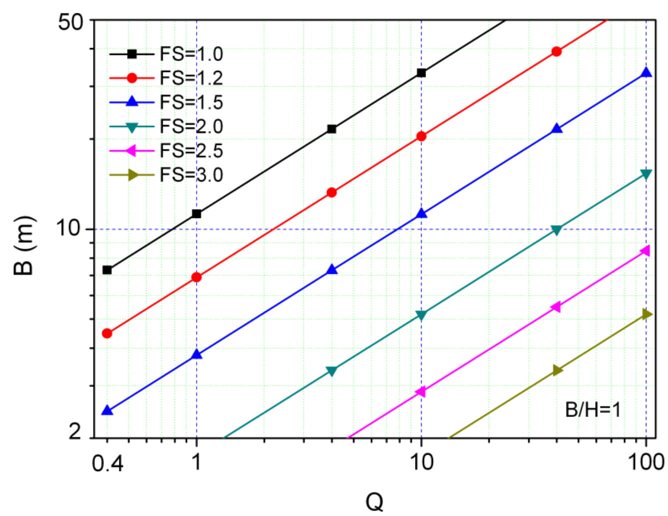


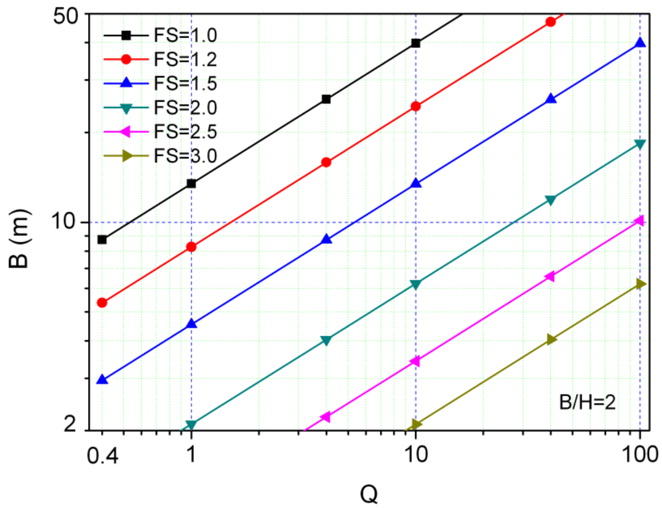
Fig. 3 Logarithmic regression results

4. Design curves of *FS*

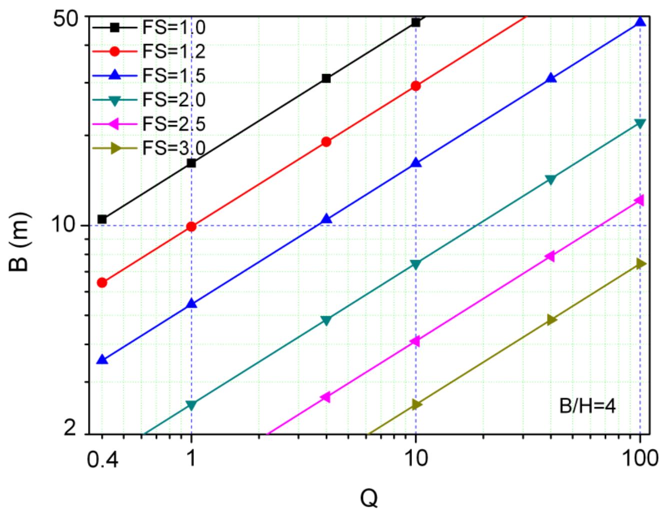
Based on the mathematical equation Eq. (4), a series of charts relating *FS* to *Q*, *B* and *H* have been developed as shown in Fig. 4. The proposed charts are potentially useful for preliminary design and checking of cavern stability.



(a)



(b)



(c)

Fig. 4 Design curves of *FS* for different *B/H* ratios: (a) *B/H*=1; (b) *B/H*=2 and (c) *B/H*=4

5. Reliability Analysis

One of the most useful software packages for probability analysis is a Microsoft Excel add-in program called @RISK (http://www.palisade.com) which can be used for Monte Carlo simulation (MCS) using sampling techniques including Latin Hypercube and Monte Carlo samplings. In this study, the derived performance function of Eq. (4) is incorporated into @RISK cells for *FS* estimation. For each MCS simulation, the number of iterations is 1000000 and Latin

Hypercube sampling is adopted. Ultimate limit state failure is deemed to occur if the predicted *FS* is greater than the critical threshold factor of safety FS_{cr}. The MCS procedures and the calculation of P_f are illustrated in Fig. 5, in which FS_{cr} is assumed as 1.0.

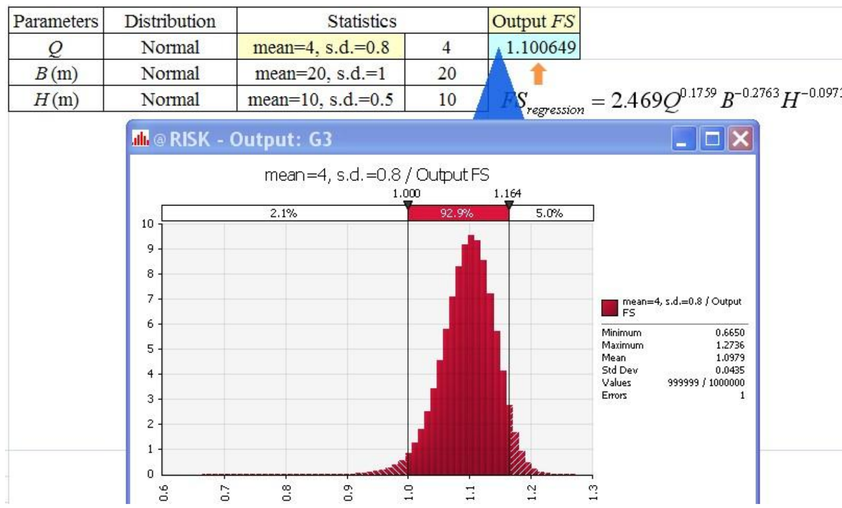


Fig. 5 MCS with @RISK

Fig. 6 shows an example analysis for B = 20 m and H = 10 m for various mean and coefficient of variation (*COV*) values of *Q*. It is clear that both the *Q* and FS_{cr} significantly influence the P_f. The results indicates that even under a high *Q* value of 10, the P_f can exceed 10% for *COV* of *Q* ≥ 0.3 and the threshold *FS* value = 1.2.

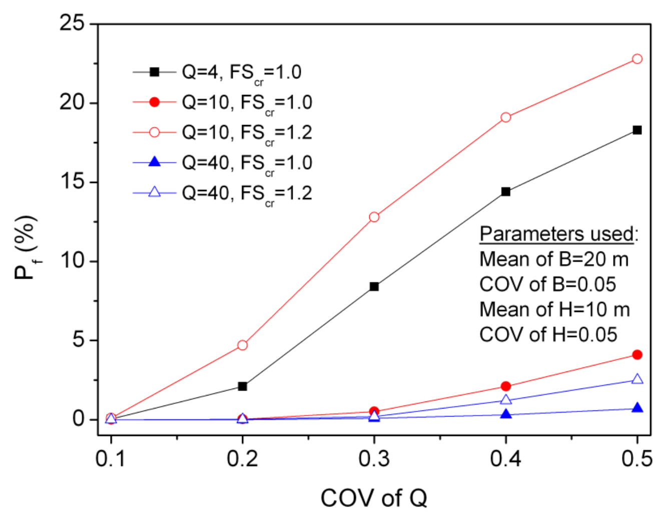


Fig. 6 Influence of *Q* and FS_{cr} on P_f

6. Summary and conclusions

Numerical analyses have been carried out using the finite difference method to assess the global stability of underground rock caverns. Analyses using logarithmic regression were used to model the relationship between *Q*, *B* and *H*, and the cavern safety factor *FS*. A comparison of the calculated and predicted *FS* values showed very good agreement with a coefficient of correlation R^2 of 0.96. Charts relating *FS* to *Q* and the cavern geometries have also been developed. The proposed charts in Fig. 4 are potentially useful for preliminary stability design and checking.

The Monte Carlo simulation using @RISK was incorporated with the logarithmic regression model to calculate the probability of failure P_f. The reliability analyses indicated that P_f is significantly influenced by *Q* and FS_{cr}. This highlights the point that a single *FS* cannot provide the complete information regarding the cavern stability as the same *FS* may indicate completely different P_f for different *COV* of *Q*. It is suggested that the *FS* and P_f be used together, as complementary measures of acceptable design.

References

[1] Bieniawski ZT. Engineering rock mass classifications. New York: Wiley and Sons; 1989.

[2] Barton N. Lien R. Lunde J. Engineering classification of rock masses for the design of tunnel support. Rock Mech 1974; 6:183–236.

[3] Meguid MA, Rowe RK. Stability of D-shaped tunnels in a Mohr-Coulomb material under anisotropic stress conditions. Can Geotech J 2006; 43:273–281.

[4] Hatzor YH, Talesnick M, Tsesarsky M. Continuous and discontinuous stability analysis of the bell-shaped caverns at Bet Guvrin, Israel. Int J Rock Mech Min Sci 2002; 39:867–886.

[5] Cundall PA. Explicit finite difference methods in geomechanics, numerical methods in engineering. In Proceedings of the EF Conference on Numerical Methods in Geomechanics, Blacksburg, Virginia 1976; 1:132–150.

[6] Shi GH. Discontinuous deformation analysis-a new model for the statics and dynamics of block systems. PhD Thesis, University of California, Berkeley; 1988.

[7] Bobet A, Fakhimi A, Johnson S, Morris J, Tonon F, Yeung M. Numerical models in discontinuous media: A review of advances for rock mechanics applications. J Geotech Geoviron 2009; 135(11): 1547–1561.

[8] Hammah RE, Yacoub T, Curran JH. Serviceability-based slope factor of safety using the shear strength reduction (SSR) method. The Second Half Century of Rock Mechanics 11th Congress of the International Society for Rock Mechanics 2007, Taylor & Francis, 1137–1140,

[9] Zhang WG, Goh ATC. Reliability assessment on ultimate and serviceability limit states and determination of critical factor of safety for underground rock caverns. Tunn Undergr Sp Tech 2012; 32: 221–230.

[10] Vermeer PA, Ruse N, Marcher T. Tunneling heading stability in drained ground. Felsbau, Jg 2002. 20(6): 8–18.

[11] Tugrul A. The application of rock mass classification systems to underground excavation in weak lime stone, Ataturk dam, Turkey. Eng Geo 1998; 50(3-4): 337–345.

[12] Bieniawski ZT. Determining rock mass deformability: experience from case histories. Int J Rock Mech Min Sci Geomech Abstr 1978; 15:237–247.

[13] Serafim JL, Pereira JP. Considerations of the geomechanics classification of Bieniawski. In Proc of Int Symp Eng Geol Underground Construction, LNEC, Lisbon, 1983; 1(II): 33–42.